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**Session 4aSP: Sensor Array Beamforming and Its Applications**

## **4aSP9. Accuracy of head-related transfer functions synthesized with spherical microphone arrays**

**Cesar D. Salvador Castaneda\*, Shuichi Sakamoto, Jorge A. Trevino Lopez, Junfeng Li, Yonghong Yan and Yoiti Suzuki**

**\*Corresponding author's address: Research Institute of Electrical Communication / Graduate School of Information Sciences, Tohoku University, 2-1-1 Katahira, Aoba-ku, Sendai, 980-8577, Miyagi, Japan, cdsalv@gmail.com**

The spherical harmonic decomposition can be applied to present realistically localized sound sources over headphones. The acoustic field, measured by a spherical microphone array, is first decomposed into a weighted sum of spherical harmonics evaluated at the microphone positions. The resulting decomposition is used to generate a set of virtual sources at various angles. The virtual sources are thus binaurally presented by applying the corresponding head-related transfer functions (HRTF). Reproduction accuracy is heavily dependent on the spatial distribution of microphones and virtual sources. Nearly-uniform sphere samplings are used in positioning the microphones so as to improve spatial accuracy. However, no previous studies have looked into the optimal arrangement for the virtual sources. We evaluate the effects of the virtual source distribution on the accuracy of the synthesized HRTF. Furthermore, our study considers the impact of spatial aliasing for a 252-channel spherical microphone array. The microphone's body is modeled as a human-head-sized rigid sphere. We evaluate the synthesis error by comparison with the target HRTF using the logarithmic spectral distance. Our study finds that 362 virtual sources, distributed on an icosahedral grid, can synthesize the HRTF in the horizontal plane up to 9 kHz with a log-spectral distance below 5 dB.

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## I. INTRODUCTION

Binaural systems can reproduce 3D auditory space with high levels of realism by including spatial hearing information. Basic perceptual cues for a listening experience arise from the scattering, reflections and resonances introduced by the pinnae and head of the listener. These phenomena can be described by the so-called head-related transfer functions (HRTFs). The HRTFs are, therefore, an important key to realizing high-definition binaural systems. However, the measurement of HRTFs is restricted to a fixed position and orientation of the head; therefore, typical binaural techniques cannot include the motion of the head in the reproduction of an auditory scene. However, it is widely known that the motion of the head can significantly improve the accuracy of spatial hearing. An alternative approach sidesteps this limitation by using an array of microphones arranged over the surface of a rigid sphere [1].

Rigid spherical arrays provide good spatial samplings of the sound pressure by using a rigid scatterer to increase the sensitivity to the arrival direction of sound. The sampled sound field can be reproduced over loudspeakers and headphones by characterizing it as a superposition of virtual sound sources surrounding the listener [2, 3]. In this context, techniques such as wave field synthesis (WFS) [4] and higher order ambisonics (HOA) [5] have been proposed for the angular interpolation and synthesis of HRTFs. These techniques use the plane-wave decomposition; therefore, they do not include distance-related effects, which may be important for rendering with high levels of realism [6].

This paper introduces a method for the angular interpolation of HRTFs considering point-like secondary sources. The use of a rigid spherical measurement surface, the size of an average human head, is proposed. The measured sound field is represented as a superposition of virtual sources for which a representative set of HRTFs has been precomputed. With the present approach, a binaural response can be synthesized from the directional distribution of the incident pressure field on the rigid sphere. Hence, this proposal is closely related to modal beamforming techniques [7, 8].

Section II briefly introduces the spherical harmonic functions used in the directional decomposition and reconstruction of sound pressure fields. They form the theoretical basis for the synthesis of HRTFs in our proposal. Section III presents the proposed method for the synthesis of high directional-resolution HRTFs in detail. Section IV shows the HRTFs on the horizontal plane synthesized using the proposed method. Concluding remarks are presented in Section V.

## II. SPHERICAL HARMONIC ANALYSIS

This section introduces the mathematical preliminaries that are necessary to decompose and reconstruct a function on the surface of the unit sphere  $\mathbb{S}^2$  in terms of spherical harmonic functions. A point in space is represented by  $\mathbf{r} = (r, \theta, \phi) = (r, \Omega)$  in the standard spherical coordinate system, where  $r$  is the radial distance,  $\theta \in [0, \pi]$  the inclination angle, and  $\phi \in [0, 2\pi]$  the azimuth angle.

The integral on the surface of the unit sphere,

$$\int_{\Omega \in \mathbb{S}^2} d\Omega = \int_0^{2\pi} \int_0^\pi \sin\theta d\theta d\phi, \quad (1)$$

covers the entire surface of  $\mathbb{S}^2$ .

The spherical harmonic functions of order  $n$  and degree  $m$  are defined by [9]

$$Y_{nm}(\Omega) = Y_{nm}(\theta, \phi) = N_{nm} P_{nm}(\cos\theta) e^{im\phi}, \quad (2)$$

where  $P_{nm}$  is the associated Legendre function and  $N_{nm}$  is a normalization factor chosen such that  $\int_{\Omega \in \mathbb{S}^2} |Y_{nm}(\Omega)|^2 d\Omega = 1$ .

The spherical harmonics are the solution to the angular part of the wave equation in spherical coordinates. They are orthonormal with respect to order  $n$  and degree  $m$ :

$$\int_{\Omega \in \mathbb{S}^2} Y_{nm}(\Omega) Y_{n'm'}^*(\Omega) d\Omega = \delta_{n-n'} \delta_{m-m'}. \quad (3)$$

The following addition theorem also holds:

$$\sum_{m=-n}^n Y_{nm}(\Omega_1) Y_{nm}^*(\Omega_2) = \frac{2n+1}{4\pi} P_n(\cos \Theta_{12}), \quad (4)$$

where  $P_n$  is the Legendre function and  $\Theta_{12}$  is the angle between the directions  $\Omega_1$  and  $\Omega_2$ .

The importance of the spherical harmonics rests in the fact that any square integrable function  $f(\Omega)$  on  $\mathbb{S}^2$  can be expanded in terms of them as follows:

$$f(\Omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^n f_{nm} Y_{nm}(\Omega), \quad (5)$$

where the constants  $f_{nm}$  define the spherical wave spectrum, a projection of  $f(\Omega)$  into the set of spherical harmonic functions computed as follows:

$$f_{nm} = \int_{\Omega \in \mathbb{S}^2} f(\Omega) Y_{nm}^*(\Omega) d\Omega. \quad (6)$$

Hence, Eqs. (5) and (6) are also referred to as the inverse spherical harmonic transform (ISHT) and the spherical harmonic transform (SHT), respectively.

### III. SYNTHESIS OF HRTFs WITH A RIGID SPHERICAL MEASUREMENT SURFACE

This section describes in detail the proposed method for the synthesis of HRTFs using a rigid spherical measurement surface and HRTFs measured for a representative set of directions.

#### Overview of the proposed method

The synthesis of high directional-resolution HRTFs using a limited number of microphones arranged on a rigid sphere of the size of an average human head is proposed. The acoustic field captured by a rigid spherical microphone array is decomposed into its directional components at the microphone positions. The decomposed acoustic field is reconstructed with a higher directional-resolution at a set of virtual source positions. The reconstructed acoustic field is then rendered binaurally with HRTFs evaluated from those virtual sources. Directional analysis is done using spherical harmonic decomposition. To compensate for the measurement effects and the propagation of the pressure field, a filter in the spherical harmonic domain needs to be computed. The compensation filter is calculated inverting the model of the acoustic scattering from the rigid sphere, assuming that the acoustic field is recorded using a continuous spherical measurement surface.

#### Scattering from the rigid sphere

Let  $\mathbf{r}_p = (r, \Omega_p)$  be a point on a spherical radiating surface of radius  $r$ , and  $\mathbf{r}_q = (a, \Omega_q)$  a point on a rigid spherical measurement surface of radius  $a$  (see Figure 1).

The presence of the rigid sphere of radius  $a$  diffracts the sound wave produced by the source at the point  $\mathbf{r}_p$ . The free field sphere-related transfer function  $S(\mathbf{r}_p, \mathbf{r}_q, k)$  is defined as the ratio

between the pressure that is actually developed at a point  $\mathbf{r}_q$  on the surface of the rigid sphere and the pressure that would be present at the center of the sphere in the free field. Thus, the interaction of sound with a rigid sphere is formulated as [10]

$$S(\mathbf{r}_p, \mathbf{r}_q, k) = \frac{-re^{-ikr}}{ka^2} \sum_{n=0}^{\infty} \frac{h_n(kr)}{h'_n(ka)} (2n+1)P_n(\cos\Theta_{pq}), \quad r > a, \quad (7)$$

where  $\Theta_{pq}$  is the angle between the source at  $\mathbf{r}_p$  and the measurement point at  $\mathbf{r}_q$ ,  $P_n$  is the Legendre function,  $h_n$  is the spherical Hankel function, and  $k$  is the wave number.

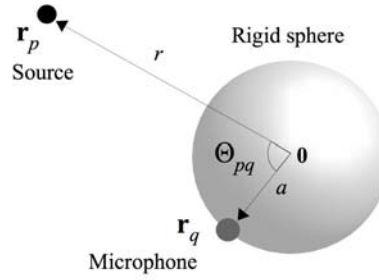


FIGURE 1: Geometry for the measurement of the sphere-related transfer function.

### Synthesis based on virtual source arrays

The high directional-resolution HRTFs, measured from sound sources at points  $\mathbf{r}_p$ , where  $p = 1, \dots, P$ , are denoted by  $H(\mathbf{r}_p, k)$ . They are synthesized by superposing the density functions  $\rho(\mathbf{r}_v, k)$  filtered with  $H(\mathbf{r}_v, k)$ , which denotes the HRTFs measured from a small set of virtual sources located at  $\mathbf{r}_v$ , where  $v = 1, \dots, V$ , for  $V < P$ . The points  $\mathbf{r}_p$  and  $\mathbf{r}_v$  are on the same spherical radiating surface of radius  $r$ . The superposition  $H_{synth}$  is defined by

$$H_{synth} = \sum_{v=1}^V H(\mathbf{r}_v, k) \rho(\mathbf{r}_v, k). \quad (8)$$

An extension of this idea to a method for the synthesis of HRTFs is to interpret  $\rho(\mathbf{r}_v, k)$  as a spherical harmonic decomposition with band-limited spherical spectrum  $\rho_{nm}(r)$ . Using Eq. (5),  $\rho(\mathbf{r}_v, k)$  can be written

$$\rho(\mathbf{r}_v, k) = \sum_{n=0}^N \sum_{m=-n}^n \rho_{nm}(r) Y_{nm}(\Omega_v), \quad (9)$$

where the truncated sum over  $n$  up to a constant  $N$  models the practical limited spatial bandwidth. Here,  $N$  must not be confused with the normalization factor  $N_{nm}$  in Eq. (2).

#### Continuous spherical measurement surface

First,  $\Omega_q$  is considered as a continuous measurement surface of radius  $a$ . The spherical wave spectrum  $\rho_{nm}(r)$  can then be computed by backpropagating the measured signal denoted by  $s(\Omega_q)$  to the radiating surface of radius  $r$ . Applying the spherical harmonic transform of Eq. (6) to  $s(\Omega_q)$ , and replacing the result in Eq. (9),  $\rho(\mathbf{r}_v, k)$  now reads

$$\rho(\mathbf{r}_v, k) = \sum_{n=0}^N \sum_{m=-n}^n B_n \int_{\Omega_q} s(\Omega_q) Y_{nm}^*(\Omega_q) d\Omega_q Y_{nm}(\Omega_v), \quad (10)$$

where the modal filter  $B_n$  must be computed so that it compensates for the measurement effects.

The measured signal can now be modeled with the acoustic scattering from the rigid sphere. Replacing  $s(\Omega_q)$  of Eq. (10) by  $S(\mathbf{r}_p, \mathbf{r}_q, k)$  of Eq. (7), and using the sum of Eq. (4), and the orthonormality property of Eq. (3) in the resulting expression,  $\rho(\mathbf{r}_v, k)$  becomes

$$\rho(\mathbf{r}_p, \mathbf{r}_v, k) = - \sum_{n=0}^N B_n \left[ \frac{h_n(kr)re^{-ikr}}{ka^2 h'_n(ka)} (2n+1) \right] P_n(\cos \Theta_{pv}), \quad (11)$$

where  $\Theta_{pv}$  is the angle between the source at  $\mathbf{r}_p$  and the virtual source at  $\mathbf{r}_v$ .

The modal filter  $B_n$  can now be chosen in such a way that it compensates the factor in brackets in Eq. (11). Therefore,

$$B_n = -(ka)^2 h'_n(ka) \times \frac{e^{ikr}}{kr(N+1)^2} \times \frac{1}{h_n(kr)}, \quad (12)$$

where its three factors, from left to right, are intended to remove the scattering effects from the rigid spherical measurement surface of radius  $a$  [1, 9, 11], to average the directional response in order to compute the pressure field at the center of the array [8], and to backpropagate the pressure field from the center of the array to the radiating surface of radius  $r$  [12].

The replacement of Eq. (12) in Eqs. (8) and (11) results in the following expression for the synthesis of HRTFs by means of a continuous spherical measurement surface:

$$H_{synth}^{cont}(\mathbf{r}_p, k) = \frac{1}{(N+1)^2} \sum_{v=1}^V H(\mathbf{r}_v, k) \sum_{n=0}^N (2n+1) P_n(\cos \Theta_{pv}). \quad (13)$$

Given that Eq. (13) is independent of the distribution of microphones, it allows the evaluation of the isolated effects arising from the arrangement of virtual sources.

#### *Discrete spherical measurement surface*

In practice, a finite number of microphones at points  $\mathbf{r}_q$ , with  $q = 1, \dots, Q$ , on the surface of the rigid sphere is used. Therefore, the integral in Eq. (10) must be approximated with a sum over  $q$ . After using the sum in Eq. (4) with the result, Eqs. (8) and (10) become the formula for the synthesis of HRTFs using a rigid spherical microphone array:

$$H_{synth}^{disc}(\mathbf{r}_p, k) = \frac{1}{4\pi} \sum_{v=1}^V H(\mathbf{r}_v, k) \sum_{n=0}^N (2n+1) B_n \sum_{q=1}^Q P_n(\cos \Theta_{vq}) c_q S(\mathbf{r}_p, \mathbf{r}_q, k), \quad (14)$$

where  $B_n$  is the same as in Eq. (12),  $\Theta_{vq}$  is the angle between the virtual source at  $\mathbf{r}_v$  and the microphone at  $\mathbf{r}_q$ , and the approximation weights  $c_q$  applied to the individual microphone signals are chosen in such way that [9]

$$\sum_{q=1}^Q c_q Y_{nm}(\Omega_q) Y_{n'm'}^*(\Omega_q) = \delta_{n-n'} \delta_{m-m'}. \quad (15)$$

#### **IV. ACCURACY ANALYSIS OF THE SYNTHESIZED HRTFS**

The use of spherical grids based on subdividing each face of an icosahedron provides a nearly regular covering of the sphere that is convenient for computations with spherical harmonic functions. When the microphones are arranged over icosahedral grids, the weights  $c_q$  in Eq. (15) can be taken to be equal with negligible error [9]. Furthermore, the maximum order  $N$  of spherical harmonics that can be approximated depends on the number of microphones:

$$(N+1)^2 \leq Q. \quad (16)$$

The number  $(N + 1)^2$  of spherical harmonics required to synthesize the HRTFs is determined by the wave number  $k$  and the radius  $a$  of the spherical scatterer. The radial spherical wave spectrum is represented by spherical Bessel functions  $j_n(kr)$ , which show a rapid decay of orders  $n > kr$ . Hence, a bound for the number of terms in the HRTF spherical harmonics expansion can be approximated by [13]

$$N = \left\lceil \frac{eka}{2} \right\rceil, \quad (17)$$

where  $e = \exp(1) \approx 2.7183$ .

To approximate the entire audible frequency range [0.02, 20] kHz using a spherical scatterer of  $a = 8.5$  cm radius, which is the size of an average human head, an order  $N = 43$  is needed, and therefore, at least  $Q = (43 + 1)^2 = 1936$  microphones over the sphere would be required. However, a limited spatial bandwidth is imposed by the reduced number of microphones in practical arrays, and hence, the accuracy can only be evaluated up to a spatial aliasing frequency.

### Evaluation conditions

The parameter under evaluation was the number of virtual sources. Its effect on the accuracy of the synthesis was evaluated. The virtual sources were arranged on the icosahedral grids described in Table 1, where the square of the division factor gives the number of subfaces. The center of the rigid spherical measurement surface was set as the reference position, and its radius was chosen to be 8.5 cm. The recorded microphone signals were generated with the model of the acoustic scattering from the rigid sphere in Eq. (7), which was computed for 360 sources at a 1.5 m distance from the reference position, equiangularly distributed on the horizontal plane. The target HRTFs from 360 sources at a 1.5 m distance equiangularly distributed on the horizontal plane were computed using the Boundary-Element-Method (BEM) [14] with a 3D mesh of the SAMRAI dummy head (see Panel A in Figure 2). Other HRTFs from sets of virtual sources at a 1.5 m distance arranged on icosahedral grids (see Table 1) were also computed with the BEM solver.

The initial evaluation of the synthesis based on a continuous spherical measurement surface was performed using Eq. (13). A second evaluation based on a discrete measurement surface was also performed. An array of  $Q = 252$  microphones distributed in a icosahedral grid over the surface of the rigid sphere of 8.5 cm radius was assumed for this purpose, which is the available setup at the Research Institute of Electrical Communication, Tohoku University [15]. The number of microphones  $Q = 252$  imposed a spatial bandwidth limitation up to the order  $N = 14$ , and therefore the accuracy could be evaluated up to a spatial aliasing frequency of around 9 kHz. The model of the acoustic scattering from the rigid sphere was decomposed up to order 14 at the positions of the microphones, and reconstructed at the positions of the virtual sources. The resultant signals were downmixed to a binaural signal. This process is formulated in Eq. (14).

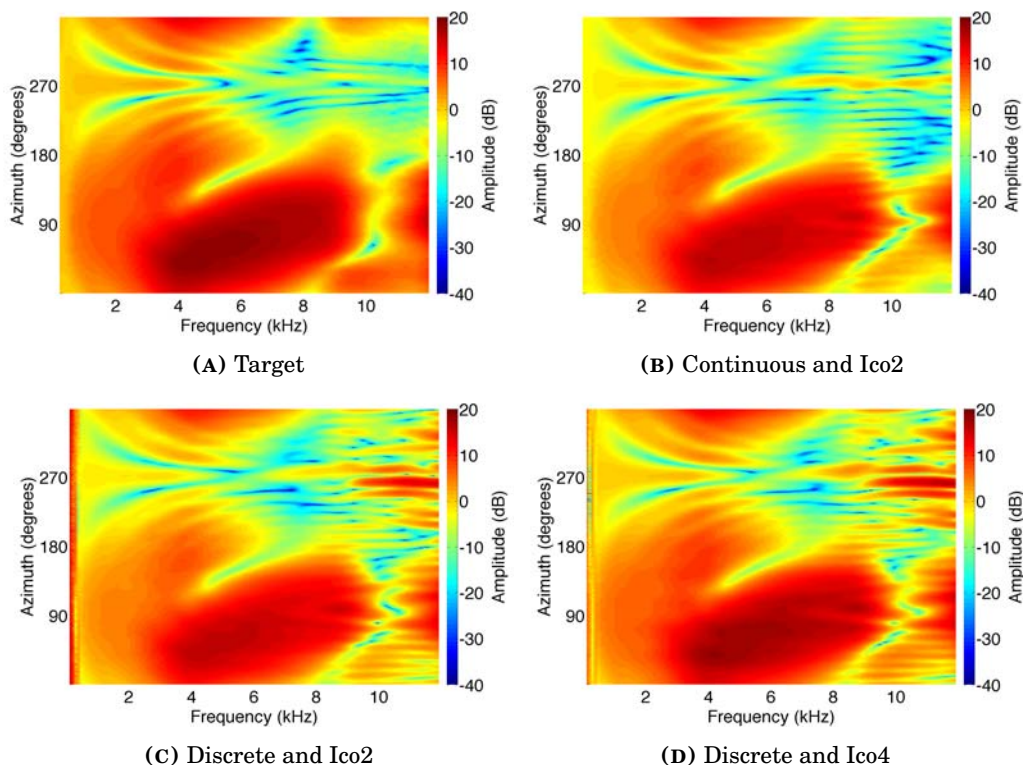
Synthesis error  $E$  was measured with the logarithmic spectral distance between the target HRTFs and the synthesized HRTFs with 252 microphones for different numbers of virtual sources distributed on icosahedral grids. It is defined by [16]

$$E = \sqrt{\frac{1}{PI} \sum_{p=1}^P \sum_{i=1}^I \left( 20 \log_{10} \left| \frac{H_{target}(\mathbf{x}_p, k_i)}{H_{synth}^{disc}(\mathbf{x}_p, k_i)} \right| \right)^2}. \quad (18)$$

The number of synthesized directions on the horizontal plane were  $P = 360$ . The number of frequency bins of the target HRTFs was 512 for a sampling frequency of 48 kHz, but the error was computed up to 9 kHz, and therefore up to  $I = 97$ .

TABLE 1: Sampling of the sphere based on subdivisions of the icosahedron

| Icosahedral grid | Division factor | Number of virtual sources |               |
|------------------|-----------------|---------------------------|---------------|
|                  |                 | in space                  | in hor. plane |
| Ico1             | 5               | 252                       | 20            |
| Ico2             | 6               | 362                       | 8             |
| Ico3             | 8               | 642                       | 12            |
| Ico4             | 10              | 1002                      | 12            |
| Ico5             | 13              | 1692                      | 2             |
| Ico6             | 16              | 2562                      | 24            |
| Ico7             | 20              | 4002                      | 36            |

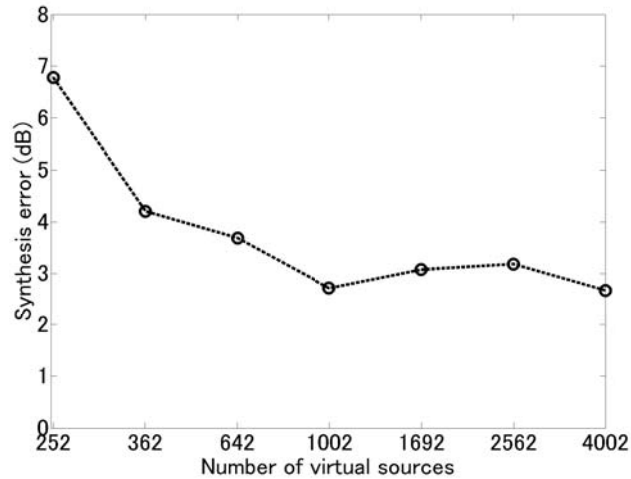


**FIGURE 2:** Synthesis of HRTFs with a rigid spherical measurement surface and virtual sources distributed on icosahedral grids. A) Target HRTFs computed with a dummy head model. B) HRTFs synthesized with a continuous measurement surface and 362 virtual sources. C) HRTFs synthesized with 252 microphones and 362 virtual sources. D) HRTFs synthesized with 252 microphones and 1002 virtual sources.

## Simulation results

Figure 2 shows the target HRTFs and the synthesized ones obtained with the proposed method for the sets Ico2 and Ico4 of virtual sources in Table 1. Panel A of Figure 2 shows the target left ear HRTFs computed with a dummy head model and sources at a 1.5 m distance on the horizontal plane. Panel B shows the HRTFs synthesized with a continuous rigid spherical measurement surface of 8.5 cm radius, spherical harmonic functions up to order 14, and 362 virtual sources distributed on an icosahedral grid. Here, the spatial aliasing effects start to appear around 8 kHz, affecting the shadowed side of the head the most. Panel C shows the HRTFs synthesized with the human-head-sized spherical array with 252 microphones and 362 virtual sources distributed on an icosahedral grid. Here, the low frequency distortion generated by the inversion of the high-order terms of the model of the acoustic scattering from

the rigid sphere appears in all directions for frequencies below 500 Hz. Panel D shows the HRTFs also synthesized with 252 microphones but now with 1002 virtual sources distributed on an icosahedral grid. Although 362 virtual sources gave an acceptable accuracy, the main benefit of increasing the number of virtual sources was the reduction of the low frequency distortion. Finally, Figure 3 shows the synthesis error for different numbers of virtual sources, where it can be seen that the lowest synthesis error was obtained using 1002 virtual sources.



**FIGURE 3:** Synthesis error measured with the logarithmic spectral distance (dB) between the target HRTFs and the synthesized HRTFs with 252 microphones for different numbers of virtual sources distributed on icosahedral grids.

## V. CONCLUSION

A method for the synthesis of high directional-resolution HRTFs using a limited number of microphones was proposed. The microphones were arranged on the surface of a spherical scatterer the size of an average human head. A limited set of HRTFs computed from virtual sources arranged over icosahedral grids around the listener was used. Spherical harmonics up to order 14 were used for the decomposition of the sound pressure field in the directions of microphones and for its reconstruction in the directions of virtual sources. A set of synthesized HRTFs on the horizontal plane, up to the spatial aliasing frequency around 9 kHz, was obtained with a logarithmic spectral distortion below 5 dB. The benefit of increasing the number of virtual sources was reduction of the low frequency distortion generated by the inversion of high order terms of the acoustic model for scattering from the rigid sphere. Ongoing work is presently being done on sources in the three-dimensional space in order to extend the evaluation to the synthesis of fully three-dimensional HRTFs.

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